Recent developments in a kinetic theory for gas-particle flows

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Abstract

Theoretical predictions are presented for the mass flux of particles transported in a turbulent gas flow based on a kinetic theory for dispersed gas-particle flows (Reeks 2021). The results depend critically upon the form of the net turbulent aerodynamic driving force which indicates that the mass flux in inhomogeneous turbulence is both diffusive and convective. Numerical calculations for particle dispersion in a kinematic simulation generated turbulent flow field indicate that this will have a significant influence on the observed build up of particle concentration in turbulent boundary layers and particle pair clustering.

Introduction

The theoretical predictions presented here for the particle mass flux in inhomogeneous turbulent flows are based on a kinetic equation (Reeks 2021) in which the net turbulent driving force acting on an elemental volume of particles (per unit particle phase space volume) is given by

$$\langle f_i W\rangle = -\left(\frac{\partial}{\partial \mathrm{x}_j}\langle W\rangle\lambda_{ji} + \frac{\partial}{\partial \mathrm{v}_j}\langle W\rangle\mu_{ji} - \kappa_i\langle W\rangle\right) \quad (1)$$

where $\boldsymbol{f}(\mathbf{x},t)$ is the force acting on an individual particle at time t. $W(\mathbf{x},\mathbf{v},t)$ is the instantaneous phase space particle number density at time t for a particle with spatial position $\mathbf{x}$ and velocity $\mathbf{v}$. $\langle ...\rangle$ refers to the average value over all realizations of the carrier flow field, so $\langle W(\mathbf{x},\mathbf{v},t)\rangle$ is the average particle phase space density or *pdf* (when normalized to one particle). See (Reeks 2021) for details on how the closure relation in Eq. (1) is derived and how it depends on the particle inertia and the Lagrangian timescales of the turbulence encountered by a particle along its trajectory in phase space. The relationship is an exact closure for Gaussian inhomogeneous carrier flow fields where λ_{ji} and μ_{ji} are the components of stress tensors and κ_i is the component of a body force per unit particle mass.

The form for the particle mass flux depends critically upon the form of the net turbulent driving force in Eq. (1) involving the divergence of the turbulent stress tensors (per unit particle mass) $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ and a body force $\boldsymbol{\kappa}$. It indicates that the mass flux is not entirely diffusive (as in homogeneous turbulence) so that in inhomogeneous turbulence, especially in near wall turbulence, there is a convective component in addition to the convection due to the body force $\boldsymbol{\kappa}$. The contributions to the total mass flux are shown more transparently in the expression for the particle mass flux $\rho\overline{\mathbf{v}}$

$$\rho\overline{\mathbf{v}} = \underbrace{\rho\left[\langle\mathbf{u}\rangle + \mathbf{v}_g + \beta^{-1}\left\{\underbrace{\left(\overline{\boldsymbol{\kappa}} - \nabla\cdot\overline{\boldsymbol{\lambda}}\right)}_{(1)} - \underbrace{\nabla\cdot\overline{\mathbf{v}'\mathbf{v}'}}_{(2)} - \underbrace{\frac{D}{Dt}\overline{\mathbf{v}}}_{(3)}\right\}\right]}_{\text{convective flux}} - \underbrace{\beta^{-1}\left(\overline{\mathbf{v}'\mathbf{v}'} + \overline{\boldsymbol{\lambda}}^{\top}\right)\cdot\nabla\rho}_{\text{diffusive flux}} \quad (2)$$

where a separation has been made into a convective flux and a diffusive flux. For completeness, the gravitational settling velocity $\mathbf{v}_g$ is included. Term (1) refers to the convective velocity derived from the net turbulent driving force as the sum of the body force (per unit particle mass) and a kinematic turbulent stress. Term (2) is the turbophoretic force (per unit particle mass) (Reeks 1983) derived from the divergence of the particle kinetic stresses. Term (3) is the inertial acceleration of the suspended particles which at equilibrium/steady-state conditions is zero or very small for weakly interacting inertial particles. Recent work has demonstrated that for gravitational settling, the kinetic approach is able to predict the increase in average settling velocity observed by particles due to interaction with structures in the turbulence, through the action of the body force contribution $\boldsymbol{\kappa}$ (Stafford & Swailes 2021). In the context of particle pair separation, where the moments of the relative velocity between particle pairs with respect to particle separation is inhomogeneous, the particle mass flux representing the mean relative velocity between particle pairs takes the same form as Eq. (2).

Numerical Simulation Results

The contributions to the mass flux have been measured as a function of particle pair separation in a kinematic simulation generated homogeneous isotropic flow field (Stafford 2020). The relative velocity between fluid point pairs is shown in Fig. 1. The fluid root mean square (rms) relative velocity as a function of pair separation r could equally well apply to the fluid rms velocity as a function of distance y^+ from the wall in a turbulent boundary layer.

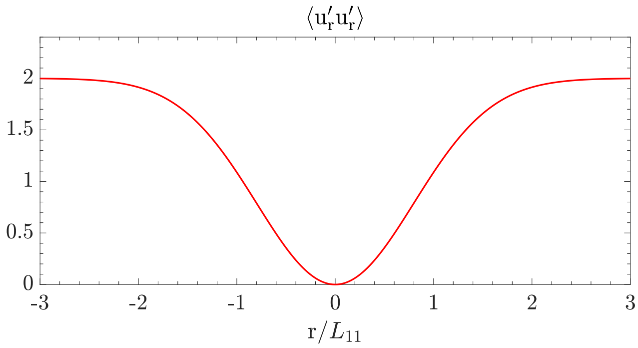


Figure 1: Mean square fluctuating flow velocity profile as a function of separation r.

In Fig. 2 the particle pair concentration ρ measured as a function of separation r is shown for different values of Stokes number St_E and average turbulence intensity u', and in all cases exhibits a build up of concentration towards the point of zero relative velocity between particle pairs.

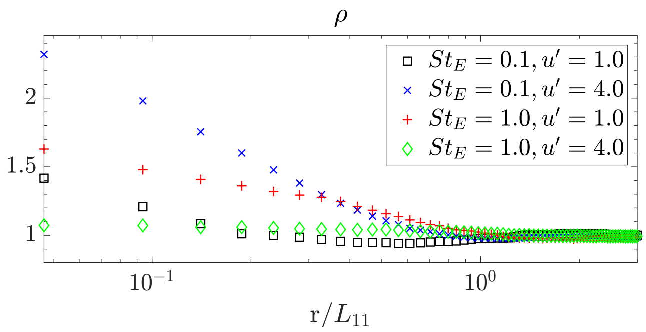
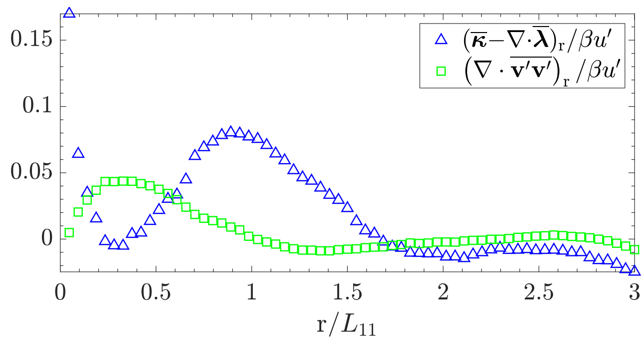
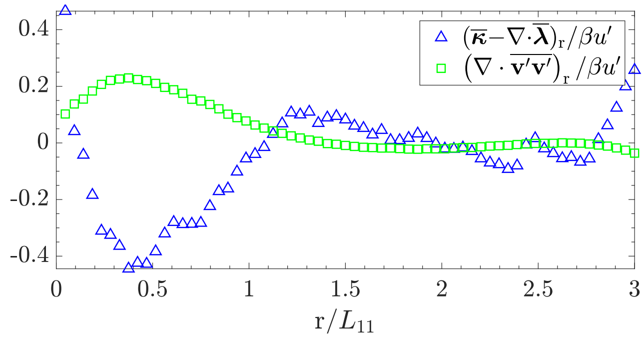


Figure 2: Normalized steady-state particle concentration.

The convective terms in the particle mass flux are shown in Fig. 3. Notably, it is observed that $\left(\overline{\boldsymbol{\kappa}} - \nabla \cdot \overline{\boldsymbol{\lambda}}\right)_{\mathrm{r}}$ is the dominant contribution for small St_E. Furthermore, the direction in which $\left(\overline{\boldsymbol{\kappa}} - \nabla \cdot \overline{\boldsymbol{\lambda}}\right)_{\mathrm{r}}$ acts varies depending upon u', becoming negative for large u' as illustrated in Fig. 3b. Since the two contributions (1) and (2) to the net convective flux act to oppose to each other within the expression for the particle mass flux in Eq. (2), the highest build up of particle concentration in Fig. 2 is accordingly seen for the case of small St_E and large u', when $\left(\overline{\boldsymbol{\kappa}} - \nabla \cdot \overline{\boldsymbol{\lambda}}\right)_{\mathrm{r}}$ becomes negative. The individual contributions that $\left(\overline{\boldsymbol{\kappa}} - \nabla \cdot \overline{\boldsymbol{\lambda}}\right)_{\mathrm{r}}$ and $\left(\nabla \cdot \overline{\mathbf{v}'\mathbf{v}'}\right)_{\mathrm{r}}$ make to the particle mass flux are then in the same direction, and act together to increase the net clustering of particles.



(a) $St_E = 0.1$, $u' = 1.0$



(b) $St_E = 0.1$, $u' = 4.0$

Figure 3: Steady-state turbulent $(\overline{\boldsymbol{\kappa}} - \nabla \cdot \overline{\boldsymbol{\lambda}})$ and kinetic $(\nabla \cdot \overline{\mathbf{v}'\mathbf{v}'})$ convective flux contributions to Eq. (2).

Conclusion

Application of the kinetic theory for turbulent dispersed gas-particle flows (Reeks 2021) shows the net turbulent aerodynamic driving force acting on suspended particles to be both diffusive and convective. Numerical simulations have demonstrated that both convective fluxes are significant to the build up of particle concentration in turbulent boundary layers and particle clustering at small particle pair separations, highlighting the importance of term (1) in Eq. (2).

References

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