A STOCHASTIC APPROACH FOR THE DEFINITION OF A NOVEL RESPONSE SPECTRUM IN URBAN AREAS

A. Tombari[(1)], P. Cacciola[(2)]

[(1)] *Senior Lecturer, University of Brighton, a.tombari@brighton.ac.uk*
[(2)] *Principal Lecturer, University of Brighton, p.cacciola@brighton.ac.uk*

Abstract

Currently, international seismic codes provide a design elastic response spectrum as a function of seismological, geological, and geotechnical parameters. These elastic response spectra are based on the prediction of the free-field motion at the ground surface, hence, by considering the propagation of the seismic waves in the soil deposit neglecting the presence of buildings nearby. However, during an earthquake, a vibrating structure emanates waves travelling through the ground over large distances that modifies significantly the energy of the free-field seismic waves in the underlying soil layers resulting in decrements of the ground motion energy in some areas and increments in others. In the urban environment, the presence of a multitude of buildings generates the occurrence of multiple interactions that are generally referred to as seismic site-city interaction.

This paper presents a study on the modification of elastic response spectra due to the influence of the urban environment. A stochastic ground motion analytical model, derived by a simplified discrete model able to capture the influence of the radiated wave field into the free field ground motion, will be used to determine the novel definition of elastic response spectrum for urban areas. Results in terms of peak structural acceleration obtained by the proposed formulation will be verified through Monte Carlo Simulation of an idealized urban area modelled through finite elements. Comparisons between the proposed and conventional elastic response spectra will be carried out on selected structures. Remarkably, this paper shows the limitations of the traditional used elastic response spectrum that might lead to an underestimation of the seismic response of structures in the urban environment. Therefore, the proposed approach presents a promising strategy to predict accurately the seismic response in urban areas offering potential modifications to the current seismic code prescriptions.

Keywords: Elastic response spectrum; urban environment; seismic site-city interaction; stochastic model;

1. Introduction

The engineering characterization of earthquake ground motion is a fundamental prerequisite of the current earthquake-resistant design of structures and infrastructures to ensure that, in event of earthquakes, the damages are limited and human lives are protected. International seismic codes such as European Eurocode 8, Italian NTC18, US ASCE 7-10, Chinese GB 50011 and Japanese BSLJ provide the earthquake motion at a given point on the surface as represented by an elastic ground acceleration response spectrum as a function of seismological, geological, and geotechnical parameters. Due to the random nature of the seismic events, the reference seismic action is only measurable in probabilistic terms through a reference probability of exceedance. No universal earthquake ground motion model are accepted yet; progresses have been made in the last few decades toward the refinement of stochastic models encompassing physical and/or seismological parameters (only to cite a few, see e.g. Deodatis, [1]; Spanos et al. [2]; Rezaeian and Der Kiureghian [3]; Cacciola and Deodatis [4]).It has to be emphasized that approaches currently proposed in the literature are used to model the free field ground motion since they are mainly based on parameters calibrated from real records obtained from instrumental stations located far away from surrounding structures. However, several studies (Kennedy et al. [5], Karatzetzou and Pitilakis, [6]) pointed out the need to incorporate additional considerations such as the effect of the soil-structure interaction for a reliable seismic design. Furthermore, during an earthquake, a vibrating building emanates waves travelling through the ground over large distances. Therefore, in the urban environment, the presence of several buildings generates multiple interactions that are generally referred to as seismic site-city interaction as shown on previous studies (see e.g. Clouteau and Aubry, [7], Kham et al., [8], Isbiliiroglu et al. [9], Wirgin [10]). Therefore, the consequent ground-motion acceleration at the free-field currently used for designing civil engineering structures can be found significantly altered within an urban area. Several methods have been used to take into account the modification of the ground motion in the urban environment in the last two-decades. Guéguen et al. [11] showed the effect of the city can be accounted for by modelling the structures as simple oscillators. Tsogka and Wirgin [12] used homogenized blocks to study the seismic response in an idealized city. Groby et al. [13] studied the seismic response of idealized 2D cities using a continuum viscoelastic medium. More recently, Cacciola and Tombari, [14] developed a novel ground motion stochastic model for the urban environment. This proposed model aims to couple the traditional ground motion stochastic models defined at the free field and analytical attenuation law models to consider the impact of vibrating structures on the surrounding free field ground motion.

In order to incorporate the novel stochastic model into the current earthquake-resistant design of structures and infrastructures, this paper proposes a definition of response spectrum in urban areas. The traditional free-field response spectrum is compared to the proposed response spectrum in presence of one or more adjacent buildings. Verification of the proposed response spectrum in urban areas is carried out by the comparison with the response spectrum determined through a Monte Carlo Simulation of a pertinent finite element model. Furthermore, parametric studies on the modification of spectral acceleration due to the presence of vibrating buildings are undertaken for various types of soil. Finally, a numerical case study for a cluster of buildings is also presented as an application to show the improvement of the prediction of the actual response spectrum determined by the use of the proposed ground motion model against the traditional approach based on the free field model.

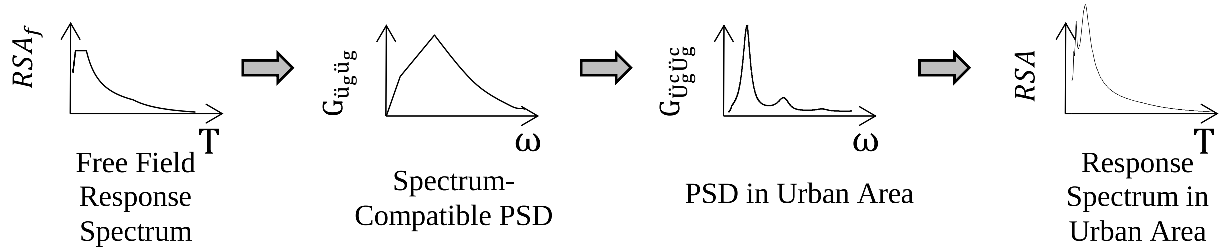

Figure 1 Proposed stochastic methodology to derive novel definition of response spectrum in urban areas.

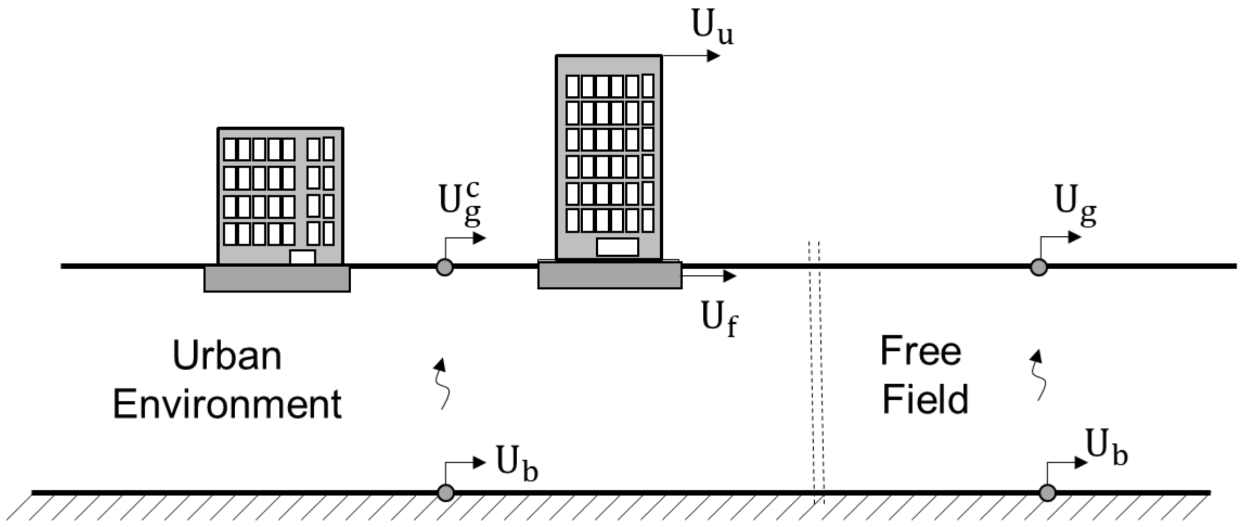

Figure 2 Seismic response of the surface ground motion in urban environment

2. Problem position

The methodology proposed in this paper modifies the free field response spectrum through a stochastic procedure in order to derive the novel definition of the response spectrum in urban areas, as depicted in Figure *1*. In this section, the stochastic ground motion model is first determined. Consider the 2D idealized portion of a city depicted in Figure *2* undergoing ground motion vibration modelled as zero mean Gaussian stationary vector process at the bedrock, $\mathbf{U}_b$, fully defined by the knowledge of the power spectral density matrix $\mathbf{G}_{ub}(\omega)$.

Under the above hypothesis the dynamic motion of the coupled urban system (after a pertinent FE discretization) is governed by the following equation in terms of absolute displacements in the frequency domain:

$$\begin{bmatrix} \mathbf{M}_u & 0 & 0 \\ 0 & \mathbf{M}_f & 0 \\ 0 & 0 & \mathbf{M}_s \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{U}}_u(\omega) \\ \ddot{\mathbf{U}}_f(\omega) \\ \ddot{\mathbf{U}}_g^c(\omega) \end{bmatrix} + \begin{bmatrix} \widetilde{\mathbf{K}}_u(\omega) & \widetilde{\mathbf{K}}_{u,f}(\omega) & 0 \\ \widetilde{\mathbf{K}}_{f,u}(\omega) & \widetilde{\mathbf{K}}_f(\omega) & \widetilde{\mathbf{K}}_{s,f}(\omega) \\ 0 & \widetilde{\mathbf{K}}_{f,s}(\omega) & \widetilde{\mathbf{K}}_s(\omega) \end{bmatrix} \begin{bmatrix} \mathbf{U}_u(\omega) \\ \mathbf{U}_f(\omega) \\ \mathbf{U}_g^c(\omega) \end{bmatrix} = \begin{bmatrix} \widetilde{\mathbf{K}}_u(\omega) & \widetilde{\mathbf{K}}_{u,f}(\omega) & 0 \\ \widetilde{\mathbf{K}}_{f,u}(\omega) & \widetilde{\mathbf{K}}_f(\omega) & \widetilde{\mathbf{K}}_{s,f}(\omega) \\ 0 & \widetilde{\mathbf{K}}_{f,s}(\omega) & \widetilde{\mathbf{K}}_s(\omega) \end{bmatrix} \mathbf{T}\mathbf{U}_b(\omega) \tag{1}$$

where $\mathbf{M}_j$,and $\mathbf{K}_j(\omega)$ are the mass and the complex stiffness submatrices in which the index $j = u, f, s$ is used for indicating the buildings superstructure in the urban environment, the building foundations and the soil, respectively. Complex-valued matrices are marked by a tilde superscript. The vector $\mathbf{U}_j$ for $j = u, f, s$ lists the displacements degrees of freedom. Note that $\mathbf{T}$ is the frequency-independent matrix given by:

$$\mathbf{T} = -Re\{\widetilde{\mathbf{K}}(\omega)\}^{-1}\mathbf{K}_b \tag{2}$$

with $\mathbf{K}_b$ is the matrix of the soil. The ground motion at a specific location within the urban environment can be, therefore, readily extracted by the response power spectral density matrix given by the following equation

$$\mathbf{G}_U(\omega) = \mathbf{H}(\omega)\, \mathbf{G}_{U_b}(\omega) \mathbf{H}^*(\omega) \tag{3}$$

where the asterisk in Eq. (3) stands for transpose complex conjugate, and the matrix $\mathbf{H}(\omega)$ is given by

$$\mathbf{H}(\omega) = (\widetilde{\mathbf{K}}(\omega) - \omega^2\mathbf{M})^{-1}\, \widetilde{\mathbf{K}}(\omega)\mathbf{T} \tag{4}$$

From Eq. (3) elements of the response power spectral density matrix $\mathbf{G}_{\mathbf{U}}(\omega)$, and in particular the elements pertinent to the degrees of freedom of the soil at the surface, are in general function of both the soil and the structures within the urban environment. Therefore, the seismic wave field on the surface is affected by the presence of the buildings that induce scattering to the ground motion waves; this produces a surface motion different from the traditionally ones used to represent the free field ground motion. Although the approach of modelling a large portion of a city might be attractive, because of the required computational demand and for the unavoidable epistemic uncertainties involved in the model, an alternative approach is thus proposed in the following sections.

2.1 Proposed solution in proximity of a vibrating building

In order to cope with the challenging task to determine the ground motion within an urban environment, in this section the simplest case considering the influence of a single vibrating structure on the nearby free-field ground motion (Figure *3*) is addressed first. Specifically, the aim is to determine a reliable ground motion model $\mathrm{U_g^c}$ able to accounting for the wave field (at a distance d) radiated by the structure undergoing ground motion excitation modelled as a stochastic process $\mathrm{U_b}$ at the bedrock. The proposed approach (see [14]) is divided into three steps: a) the solution of dynamic response of the soil-structure interaction system; b) the evaluation of the radiated wave field generated by the vibrating structure and c) the evaluation of the ground motion in the proximity of a vibrating structure as a superposition of the free field ground motion and the radiated wave field. The model of Figure *3* comprises a SDOF superstructure characterized by structural stiffness, $\tilde{\mathrm{k}}_{\mathrm{str}}$, a mass at the top of the superstructure, m_{str}, a mass at foundation level, $\mathrm{m_f}$, and the foundation-soil system that can be fully defined by the soil-structure interaction impedance, $\tilde{\mathrm{k}}_{\mathrm{SSI}}(\omega)$. In this paper, only the horizontal absolute components of the structure and foundation displacements, U and $\mathrm{U_f}$, respectively, are considered. The equation governing the motion of the system in terms of absolute displacements in the frequency domain reads

$$(\widetilde{\mathbf{K}}(\omega) - \omega^2\mathbf{M})\mathbf{U}(\omega) = \widetilde{\mathbf{K}}(\omega)\boldsymbol{\tau}\mathrm{U_{FIM}}(\omega) \tag{5}$$

where $\mathrm{U_{FIM}}(\omega)$ is foundation input motion, namely the motion of the massless foundation under seismic loading, and $\boldsymbol{\tau}$ is the influence vector.

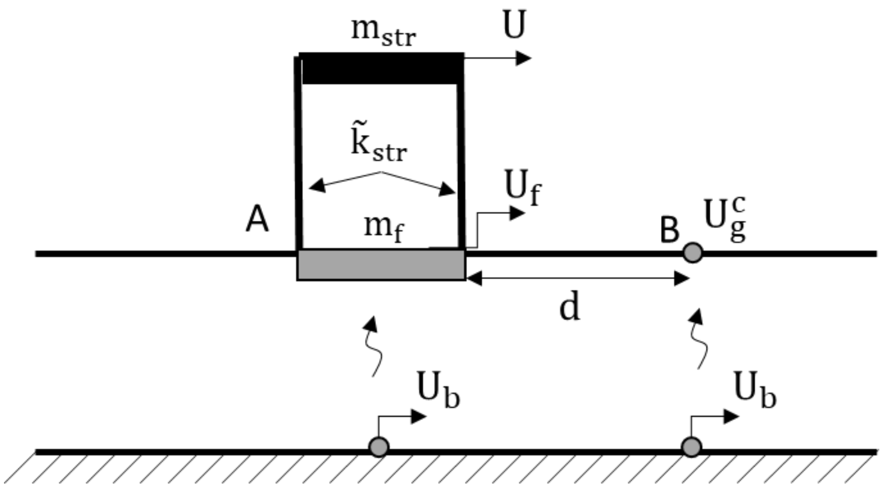

Figure 3 Seismic wave field surrounding a building induced by bedrock excitation

Therefore, from Eq. (5), the solution in its expanded form is given by

$$\begin{bmatrix} U(\omega) \\ U_f(\omega) \end{bmatrix} = \left(\begin{bmatrix} \tilde{k}_{str} & -\tilde{k}_{str} \\ -\tilde{k}_{str} & \tilde{k}_{str} + \tilde{k}_{SSI}(\omega) \end{bmatrix} - \omega^2 \begin{bmatrix} m_{str} & 0 \\ 0 & m_f \end{bmatrix} \right)^{-1} \begin{bmatrix} 0 \\ \tilde{k}_{SSI}(\omega) \end{bmatrix} U_{FIM}(\omega) \quad (6)$$

where, the complex-valued nature of the components is associated to the adoption of the hysteretic damping, $\tilde{k}_{str} = k_{str}(1 + i\eta)$, i is the imaginary unit and η is the loss factor. Furthermore, the dependence of $\tilde{k}_{SSI}$ from the circular frequency ω is hereinafter omitted for simplicity sake and determined through a static approach, without affecting the generality of the formulation. Therefore, the frequency transfer function of the foundation displacement, $H_f(\omega)$, defined as the ratio between the foundation displacement, $U_f(\omega)$, and the foundation input motion, $U_{FIM}(\omega)$, is readily derived as follows:

$$H_f(\omega) = \frac{U_f(\omega)}{U_{FIM}(\omega)} = \frac{\tilde{\omega}_f^2(\omega^2 - \tilde{\omega}_0^2)}{\omega^4 - \omega^2(\tilde{\omega}_0^2 + \tilde{\omega}_f^2) - \tilde{\omega}_0^2(\beta\omega^2 - \tilde{\omega}_f^2)} \quad (7)$$

where

$$\omega_0^2 = \left(\frac{k_{str}}{m_{str}}\right) \quad (8)$$

is the squared circular natural frequency of the fixed base SDOF superstructure and,

$$\omega_f^2 = \left(\frac{k_{SSI}}{m_f}\right) \quad (9)$$

is the squared circular natural frequency of the soil-foundation system, and

$$\beta = \left(\frac{m_{str}}{m_f}\right) \quad (10)$$

Once $U_f(\omega)$ is determined, the wave field radiated by the vibrating foundation needs to be determined. It is noted that the radiated wave field depends from the geometry of the foundation and different numerical strategies can be adopted (see e.g. [15],[16], and [17]). With the aim of developing an analytical model, in this paper, the foundation of the structure, assumed shallow, is approximated by an equivalent cylindrical shape. As a consequence, the asymptotic cylindrical waves propagating from a cylinder subjected to a harmonic signal can be determined through the attenuation function $\alpha(d, \omega)$, given by [18]:

$$\alpha(d, \omega) = \sqrt{\frac{a}{d}} \exp\left(-\frac{\eta_g \omega d}{V}\right) \exp\left[-i\omega\left(\frac{d}{V}\right)\right] \qquad \forall d \geq a \quad (11)$$

where a is the equivalent radius of the foundation of the equivalent oscillator, d is the distance between a selected point on the ground surface and the border of the foundation and V is the velocity of propagation of the waves through the soil having Poisson ratio, ν, and soil loss factor, η_g. In the ground surface plane, Eq. (11) can be decomposed into two orthogonal components, as done in Poulos, [19], as follows:

$$\alpha(d, \omega) = \alpha_{\parallel}(d, \omega) \cos^2\theta + \alpha_{\perp}(d, \omega) \sin^2\theta \qquad \forall d \geq a \quad (12)$$

where θ is the angle of the line connecting source and receiver, $\alpha_{\parallel}$ and $\alpha_{\perp}$ are the components parallel and perpendicular to the direction of propagation of the dynamic input, respectively. Approximate expressions of $\alpha_{\parallel}$ and $\alpha_{\perp}$ can be found in Dobry and Gazetas, [20] as follows

$$\alpha_{\perp}(d, \omega) = \sqrt{\frac{a}{d}} \exp\left(-\frac{\eta_g \omega d}{V_s}\right) \exp\left[-i\omega\left(\frac{d}{V_s}\right)\right] \qquad \forall d \geq a \quad (13)$$

in which $V_s = \sqrt{G/\rho}$ is the shear wave velocity of the soil with shear modulus G and mass density ρ, and

$$\alpha_{\parallel}(d, \omega) = \sqrt{\frac{a}{d}} \exp\left(-\frac{\eta_g \omega d}{V_{La}}\right) \exp\left[-i\omega\left(\frac{d}{V_{La}}\right)\right] \qquad \forall d \geq a \quad (14)$$

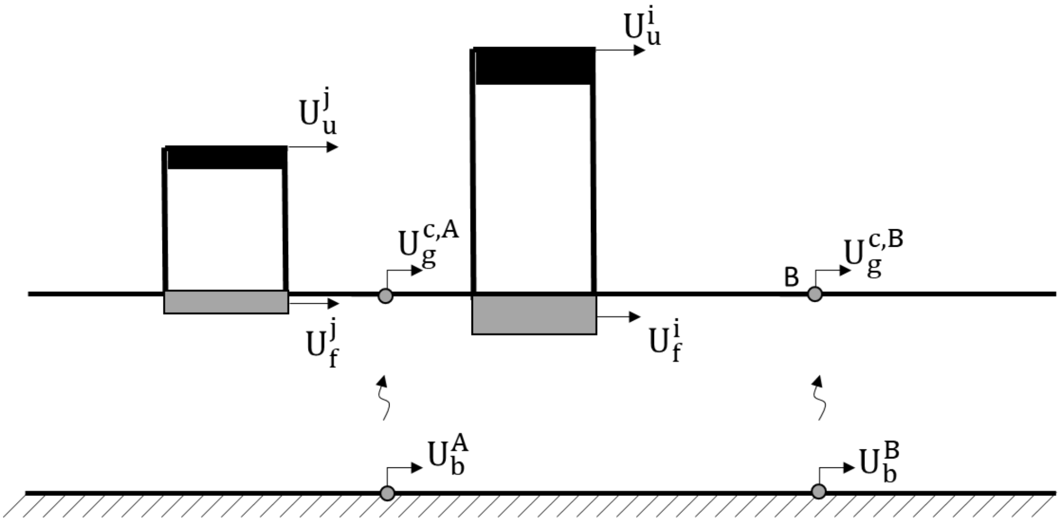

Figure 4 Seismic wave field surrounding a cluster of buildings induced by coherent or incoherent bedrock excitation

where $V_{La} = (3.4V_s)/[\pi(1-\nu)]$ is the Lysmer's analogue velocity. Therefore the wave field radiated by the vibrating foundation is given by

$$U_g^f(d,\omega) = \alpha(d,\omega)U_f^r(\omega) = \sqrt{\frac{a}{d}}\exp\left(-\frac{\eta\omega d}{V_{La}}\right)\exp\left[-i\omega\left(\frac{d}{V_{La}}\right)\right]U_f^r(\omega) \quad \forall d \geq a \tag{15}$$

where $U_f^r(\omega) = U_f(\omega) - U_{FIM}(\omega)$. Finally, at ground level in proximity of the vibrating structure at a certain distance d, $U_g^c(d,\omega)$, induced by the motion of the foundation $U_f(\omega)$ is determined as a superposition of the two effects, namely: the free field motion, $U_g(\omega)$, and the radiated motion $U_g^f(d,\omega)$, that is

$$U_g^c(d,\omega) = U_g(\omega) + U_g^f(d,\omega) \tag{16}$$

or equivalently:

$$U_g^c(d,\omega) = U_g(\omega) + \alpha(d,\omega)\big(U_f(\omega) - U_{FIM}(\omega)\big) \tag{17}$$

Eqs (16) and (17) represent the ground motion in proximity of a vibrating structure at the soil surface and require the knowledge of the free field ground motion (in absence of buildings) and the foundation response.

2.2 Proposed Stochastic Ground Motion Model for the Urban Environment

In this section, the previous stochastic model is extended in order to take into account the contribution of multiple vibrating structures on the surface exploiting the linearity of the problem. Specifically, consider the case depicted in Figure *4*, in which two buildings, are represented along with the relevant degrees of freedom. Following Poulos's assumption (Poulos, [19], Dobry and Gazetas, [20]) only the interaction of one source and one receiver at a time is considered. Therefore, the interaction between two building foundations is neglected. This assumption leads directly to the following representation of the ground motion process at selected point within the urban environment (see Eqs. 16 and 17):

$$U_g^{c,k}(\omega) = U_g^k(\omega) + \sum_{i=1}^{n}\alpha_i(d_{ik},\omega)\left(U_f^k(\omega) - U_g^k(\omega)\right) \qquad k = A, B, \ldots \tag{18}$$

with $U_f^k(\omega)$ given by

$$U_f^k(\omega) = H_f^k(\omega)U_g^k(\omega) \tag{19}$$

where $\mathrm{U}_{\mathrm{g}}^{\mathrm{c,k}}$ is the modified ground motion process at the location k, $\mathrm{U}_{\mathrm{g}}^{\mathrm{k}}(\omega)$ is the free field ground motion process, $\mathrm{H}_{\mathrm{f}}^{\mathrm{k}}(\omega)$ is the foundation transfer function, $\mathrm{U}_{\mathrm{f}}^{\mathrm{k}}(\omega)$ is the foundation response, $\alpha_i(d_{ik},\omega)$ is the attenuation function radiated by the ith structure at a distance d_{ik} between the foundation of the ith structure and the selected point k, and n is the number of structures considered. Therefore, Eq. (18) can be conveniently rewritten in the following matrix form

$$\mathrm{U}_{\mathrm{g}}^{\mathrm{c}}(\omega)=\mathbf{H}_{\alpha}(\omega)\mathbf{U}_{\mathrm{g}}\,(\omega) \tag{20}$$

where $\mathbf{H}_{\alpha}(\omega)$ is the following $(\mathrm{n}+1)\times 1$ vector

$$\mathbf{H}_{\alpha}(\omega)=[\Psi_1(d_{1k},\omega)\quad\cdots\quad\Psi_{\mathrm{n}}(d_{nk},\omega)\quad 1] \tag{21}$$

where $\Psi_{\mathrm{i}}(d_{ik},\omega)=\alpha_{\mathrm{i}}(d_{ik},\omega)\mathrm{H}_{\mathrm{f}}^{\mathrm{r,i}}(\omega)$ and $\mathrm{H}_{\mathrm{f}}^{\mathrm{r,i}}(\omega)=\mathrm{H}_{\mathrm{f}}^{\mathrm{i}}(\omega)-1$. Therefore, the power spectral density function of the ground motion in the urban environment altered by the presence of *n* structures assumes the following form

$$\mathrm{G}_{\ddot{\mathrm{U}}_{\mathrm{g}}^{\mathrm{c}}\ddot{\mathrm{U}}_{\mathrm{g}}^{\mathrm{c}}}(\omega)=\mathbf{H}_{\alpha}(\omega)\mathbf{G}_{\ddot{\mathrm{u}}_{\mathrm{g}}\ddot{\mathrm{u}}_{\mathrm{g}}}(\omega)\mathbf{H}_{\alpha}(\omega)^{*} \tag{22}$$

Clearly the *n* structures to include model defined by Eq. (22) are only the structures nearby the selected point. The preliminary study of the attenuation functions will determine the size of the problem. After simple algebra it can be shown that for fully coherent Gaussian stationary ground motion process the power spectral density function of the ground motion in the selected point *k* on the soil surface is given by

$$\mathrm{G}_{\ddot{\mathrm{U}}_{\mathrm{g}}^{\mathrm{c}}\ddot{\mathrm{U}}_{\mathrm{g}}^{\mathrm{c}}}(\omega)=\left|1+\textstyle\sum_{\mathrm{i}=1}^{\mathrm{n}}\Psi^{\mathrm{i}}(d_{ik},\omega)\right|^2\mathrm{G}_{\ddot{\mathrm{U}}_{\mathrm{g}}\ddot{\mathrm{U}}_{\mathrm{g}}}(\omega) \tag{23}$$

that can be readily applied in any stochastic analysis.

3. Response Spectrum in Urban Areas

Finally, the novel definition of response spectrum in urban areas is here derived. The spectral acceleration is obtained by using the random vibration theory as the median value of the largest peak of the response of each single linear oscillator system (SDOF) subjected to the ground motion in urban areas described by Eq. (23). Therefore, the response spectrum in acceleration (RSA) as a function of the period, T, can be derived as follows:

$$\mathrm{RSA(T)}=\omega_0^2\eta_{\mathrm{U}_{\mathrm{SDOF}}^{\mathrm{c}}}\left(\mathrm{T_W},\lambda_{0,\mathrm{U}_{\mathrm{SDOF}}^{\mathrm{c}}},\lambda_{1,\mathrm{U}_{\mathrm{SDOF}}^{\mathrm{c}}},\lambda_{2,\mathrm{U}_{\mathrm{SDOF}}^{\mathrm{c}}}\right)\sqrt{\lambda_{0,\mathrm{U}_{\mathrm{SDOF}}^{\mathrm{c}}}} \tag{24}$$

where $\mathrm{T_W}$ is the time observing window; $\lambda_{i,\mathrm{U}_{\mathrm{SDOF}}^{\mathrm{c}}}$ $(i=0,1,2)$ are the ith-order response spectral moments of the response of the SDOF system and $\eta_{\mathrm{U}_{\mathrm{SDOF}}^{\mathrm{c}}}$ is the peak factor (see [21]) given by

$$\eta_{\mathrm{U}_{\mathrm{SDOF}}^{\mathrm{c}}}=\sqrt{2\ln\left\{2N_{\ddot{\mathrm{U}}_{\mathrm{SDOF}}^{\mathrm{c}}}\left[1-\exp\left[-\delta_{\ddot{\mathrm{U}}_{\mathrm{SDOF}}^{\mathrm{c}}}^{1.2}\sqrt{\pi\ln\left(2N_{\mathrm{U}_{\mathrm{SDOF}}^{\mathrm{c}}}\right)}\right]\right]\right\}} \tag{25}$$

with

$$N_{\mathrm{U}_{\mathrm{SDOF}}^{\mathrm{c}}}=\frac{T_W}{-2\pi\ln 0.5}\sqrt{\frac{\lambda_{2,\mathrm{U}_{\mathrm{SDOF}}^{\mathrm{c}}}}{\lambda_{0,\mathrm{U}_{\mathrm{SDOF}}^{\mathrm{c}}}}} \tag{26}$$

and

$$\delta_{\mathrm{U}_{\mathrm{SDOF}}^{\mathrm{c}}}=\sqrt{1-\frac{\lambda_{1,\mathrm{U}_{\mathrm{SDOF}}^{\mathrm{c}}}^2}{\lambda_{0,\mathrm{U}_{\mathrm{SDOF}}^{\mathrm{c}}}\lambda_{2,\mathrm{U}_{\mathrm{SDOF}}^{\mathrm{c}}}}} \tag{27}$$

where the response spectral moments $\lambda_{i,U^c_{SDOF}}$ are given by the following equation:

$$\lambda_{i,U^c_{SDOF}} = \int_0^{+\infty} \omega^i G_{U^c_{SDOF}}(\omega) d\omega \tag{28}$$

with $G_{U^c_{SDOF}}(\omega)$ power spectral density function of the response of SDOF system. The response of the SDOF system in urban areas can be obtained throug the following equation:

$$G_{U^c_{SDOF}} = |H_{SDOF}(\omega)|^2 G_{U^c_g U^c_g} \tag{29}$$

where the transfer function of the SDOF system is:

$$H_{SDOF}(\omega) = \frac{1}{(1-\Omega^2)+i\eta\Omega} \tag{30}$$

In which $\Omega = {}^{\omega}/_{\omega_0} = {}^{\omega T}/_{2\pi}$, for each natural period, T, of the considered SDOF system.

4. Numerical Results

4.1 Parametric Analysis

The proposed methodology to determine the modification of the response spectrum is applied in this section. Pertinent data are reported in Table *1*. Figure *5* shows the response spectra for three soil types, i.e. B (Figure *5*a), C (Figure *5*b), and D (Figure *5*c), according to EN1998-1, for several distances. The traditional response spectrum for free field motion is indicated through a grey shade. It can be observed that the response around the vibrating building modifies the traditional response by generating zones with higher and lower spectrum acceleration. Remarkably, the influence of vibrating structure for the selected set of structural parameters significantly increases the value of the response spectrum at certain periods. Specifically, Figures 5d-e-f show the proposed response spectrum at a distance equal to 10m from the vibrating building for the three soil types B-C-D, respectively. It is worth noting that a peak of the spectral acceleration is achieved around the fundamental period of the vibrating building considering soil-structure interaction effects; therefore, the peak occurs at longer periods with the decrease of the shear wave velocities of the soil. Moreover, these effects caused by the propagation of the waves generated by the vibrating building through the soil, are detected at long distances of 100m in stiff soils.

4.2 Comparison of the proposed analytical model with MCS

In this section the proposed response spectrum model around a single vibrating structure is compared with the results obtained by applying the Monte Carlo Simulation of the pertinent finite element model depicted in Figure 6. The geotechnical and structural data used to characterize the model are reported in Table *1* for ground types B, C, and D according to EN1998-1. The finite element soil domain is 50m-deep and 800m-wide in order to avoid reflections of the waves on the lateral free boundaries of the domain. The soil domain is modelled with 4-Node Quadrilateral Elements under plane strain conditions. A constant modal damping equal to 0.05 is considered for both soil and structure, corresponding at the hysteric damping with loss factor of 0.1 at the fundamental periods of the model. Seismic excitation is applied at the bedrock of the soil deposit. Monte Carlo simulation is performed by considering 100 Gaussian stationary sample functions of spectrum-compatible ground motion time histories (see [4]).

Table 1 Parameters used to characterize the response spectrum for ground type B – C – D

ω_0	β	${}^{\omega_f}/_{\omega_0}$	a	V_s	η
31.42 rad/s	2	1.2(B)-0.63(C)-0.31(D)	3m	400(B)-200(C)-100(D)	0.1

The average response spectrum acceleration (RSA) is obtained at two locations of the surface, i.e. point A ($s = 10\text{m}$) and B ($s = 20\text{m}$) and compared with the proposed response spectrum in Figure *7*. It can be observed a good matching between the proposed RSA, indicated by a black continuous curve with the average response spectrum derived from the MCS, shown as a dashed black curve, especially around the natural frequency of the structure considering soil-structure interaction effects. As observed in the previous parametric analysis, the response spectrum around a building is largely different from the response spectrum defined in the seismic codes, such as EN1998-1 represented by the continuous blue curve. Moreover, on the same Figure *7*, response spectrum derived from Cacciola [22] and average response spectrum on the far field are depicted to verify the numerical model against the analytical models.

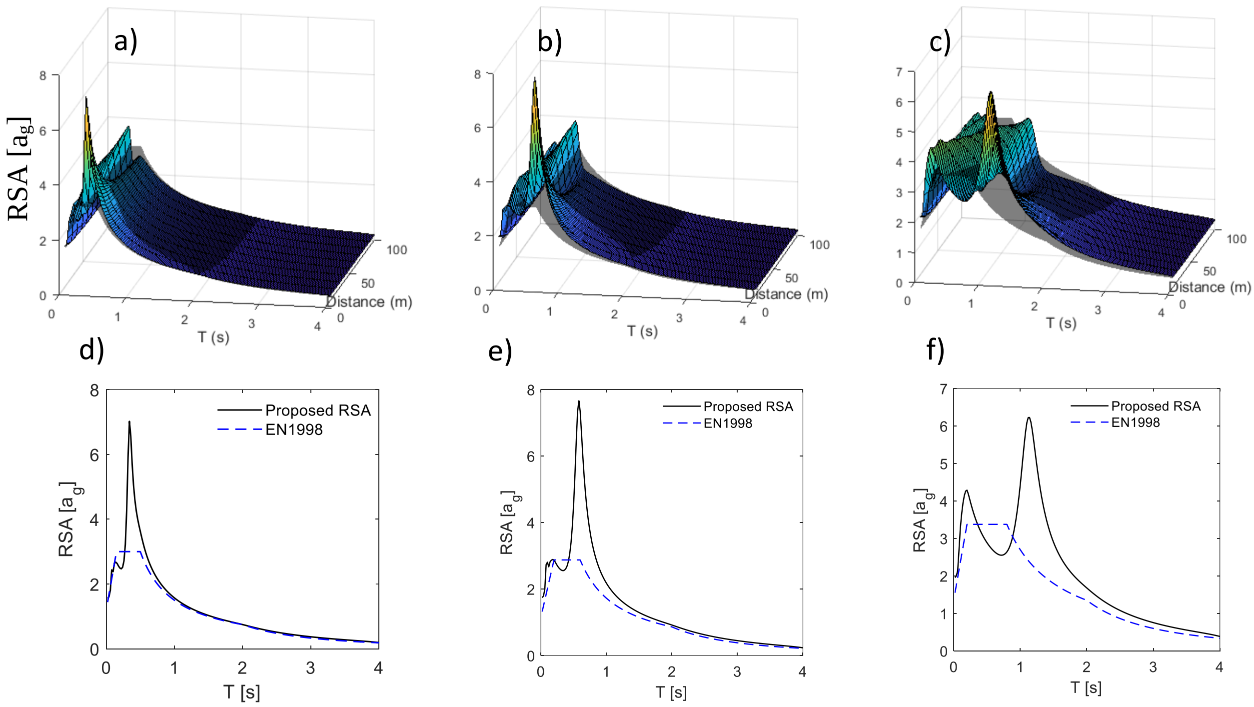

Figure 5 Response Spectra around a vibrating structure compared to free field motion response spectrum compatible to ground types B – C – D according EN 1998-1, for varying distances (a-b-c) and at distance equal to 10 m (d-e-f), respectively.

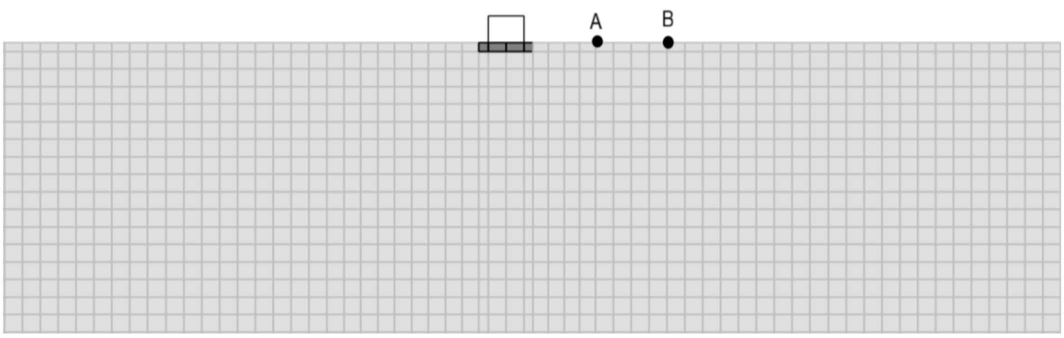

Figure 6 Close up sketch of the finite element model used to calculate the response spectrum in urban enviroment in two distances A and B.

4.3 Application to a cluster of buildings

The influence of a cluster of buildings depicted in Figure *8*a on the free field motion is assessed through the Eqs. (23-24). The complex of buildings comprises 6 reinforced concrete frame structures, which parameters used to identify the ground motion model are reported in Table *2*. Figure *9*a shows the proposed response spectrum obtained considering the individual building B1 of Figure *8*b through Eq. (24) compared to the response spectrum according to EN1998-1. Note that there are clearly identifiable zones with higher peak response than the traditional free-field response spectrum, manifesting the underestimation of the seismic forces determined by using the current response spectra. The proposed response spectrum is well in agreement with the average response spectrum obtained from the Monte Carlo Simulation analysis, indicated by a dashed curve; the mismatch occurs at one peak induced by the 2nd mode of vibration of the building that disappears when the analysis is limited to the first mode, as indicated by the dashed red curve. Figure *9*b shows the proposed response spectrum obtained also considering the buildings adjacent to B1 by adopting Eq. (24). The presence of more than one vibrating buildings affect the overall response at the location A, generating higher spectral accelerations than what obtained earlier.

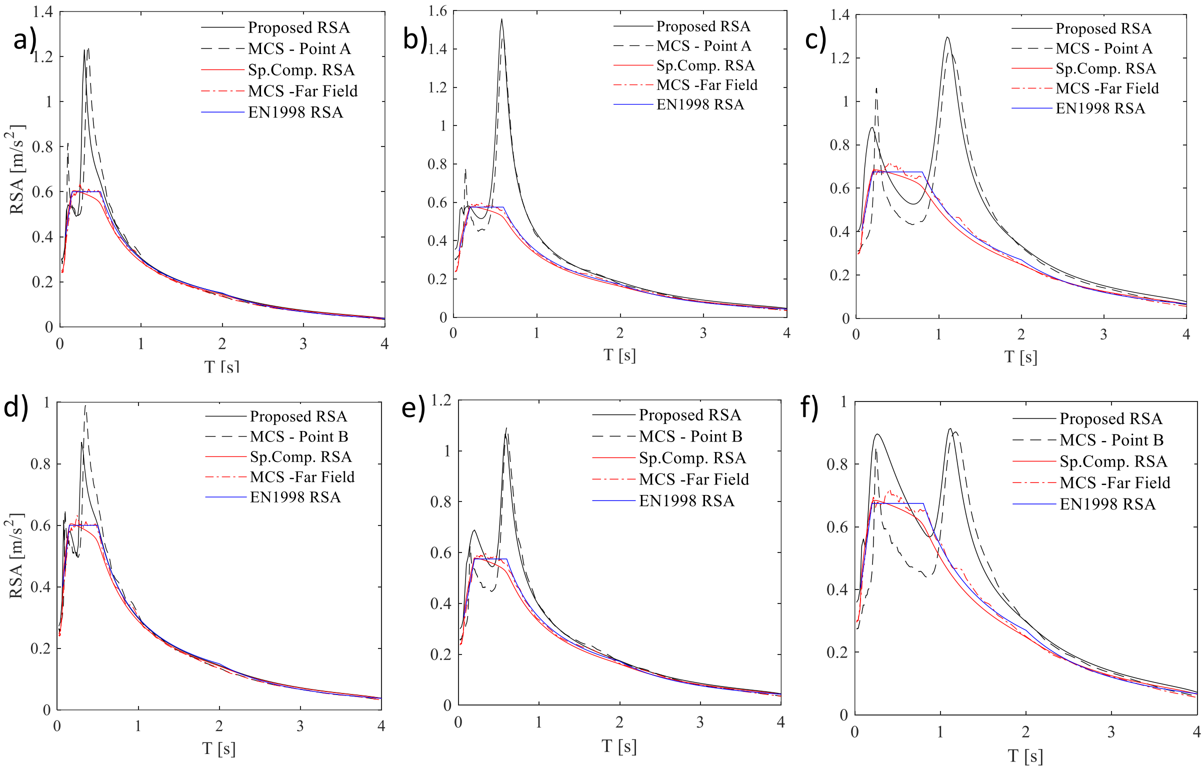

Figure 7 Proposed response spectra for ground types B (a-d), C (b-e), and D (c-f) at location A and B, respectively.

Table 2 Main data used to define the ground motion model for the cluster of buildings

	B1	B2	B3	B4	B5	B6
ω_0 [rad/s]	11.81	11.32	13.95	13.95	16.96	11.32
ω_F [rad/s]	147	91.90	85.00	93.00	85.00	91.90
d - A [m]	10	57	20	26	35	76

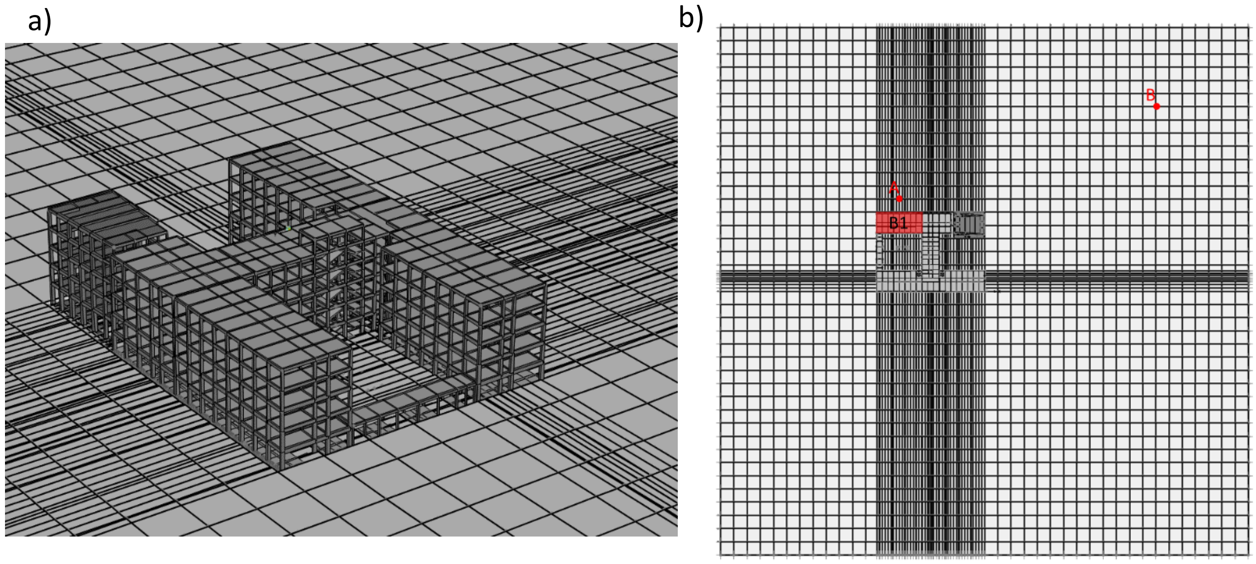

Figure 8 Complex of buildings a) investigated and b) position of the building B1 related to considered point A and B.

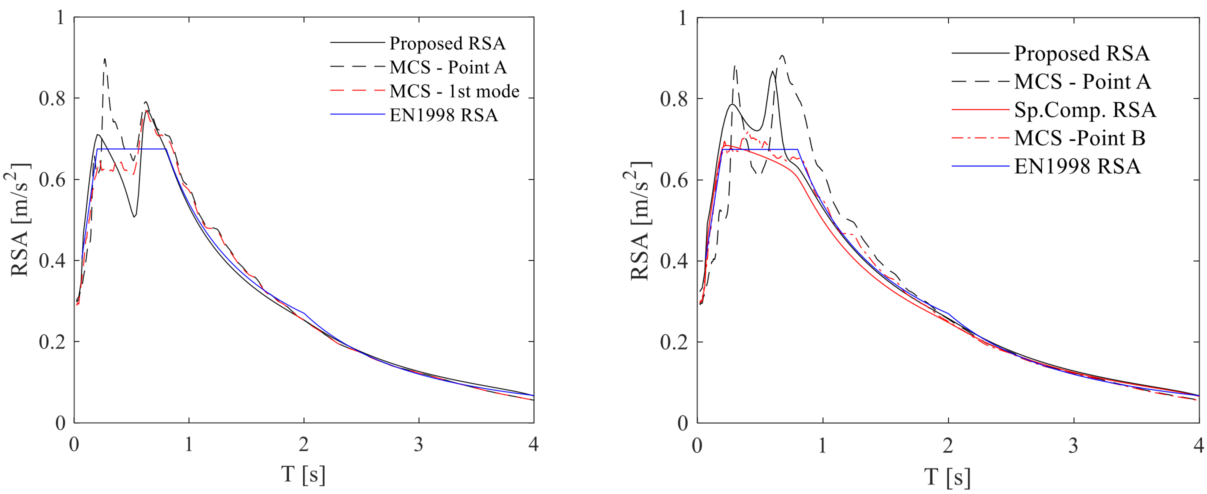

Figure 9 Response spectra considering a) only building B1 and b) the cluster of buildings at the locations A-B

5. Concluding Remarks

This paper presents a first attempt to study the modification of elastic response spectra able due to the influence of the urban environment for the seismic structural design. A stochastic ground motion analytical model, derived by a simplified discrete model able to capture the influence of the radiated wave field into the free field ground motion, has been used to determine the novel definition of elastic response spectrum for urban areas. The proposed methodology to determine the response spectrum in urban areas has been validated through Monte Carlo Simulations of finite element models. Remarkably, limitations of the traditional used elastic response spectrum that lead to an underestimation of the seismic response of structures in the urban environment, has been shown. Therefore, the proposed approach presents a promising strategy to predict accurately the seismic response in urban areas offering potential modifications to the current seismic code prescriptions.

6. References

[1] G. Deodatis, ‘Non-stationary stochastic vector processes: seismic ground motion applications’, *Probabilistic Engineering Mechanics*, vol. 11, no. 3, pp. 149–167, Jul. 1996, doi: 10.1016/0266-8920(96)00007-0.

[2] P. D. Spanos, A. Giaralis, and N. P. Politis, ‘Time–frequency representation of earthquake accelerograms and inelastic structural response records using the adaptive chirplet decomposition and empirical mode decomposition’, *Soil Dynamics and Earthquake Engineering*, vol. 27, no. 7, pp. 675–689, Jul. 2007, doi: 10.1016/j.soildyn.2006.11.007.

[3] S. Rezaeian and A. Der Kiureghian, ‘Simulation of synthetic ground motions for specified earthquake and site characteristics’, *Earthquake Engineering & Structural Dynamics*, p. n/a-n/a, 2010, doi: 10.1002/eqe.997.

[4] P. Cacciola and G. Deodatis, ‘A method for generating fully non-stationary and spectrum-compatible ground motion vector processes’, *Soil Dynamics and Earthquake Engineering*, vol. 31, no. 3, pp. 351–360, Mar. 2011.

[5] R. P. Kennedy, R. H. Kincaid, and S. A. Short, ‘Engineering characterization of ground motion Task II Effects of ground motion characteristics on structural response considering localized structural nonlinearities and soil-structure interaction effects Volume 2’, NTIS, United States, NUREG/CR--3805-Vol2, 1985.

[6] A. Karatzetzou and D. Pitilakis, ‘Modification of Dynamic Foundation Response Due to Soil-Structure Interaction’, *Journal of Earthquake Engineering*, vol. 22, no. 5, pp. 861–880, May 2018, doi: 10.1080/13632469.2016.1264335.

[7] D. Clouteau and D. Aubry, ‘Modifications of the Ground Motion in Dense Urban Areas’, *Journal of Computational Acoustics*, vol. 9, no. 4, pp. 1659–1675, Dec. 2001, doi: 10.1016/S0218-396X(01)00150-9.

[8] M. Kham, J.-F. Semblat, P.-Y. Bard, and P. Dangla, ‘Seismic Site-City Interaction: Main Governing Phenomena through Simplified Numerical Models’, *Bulletin of the Seismological Society of America*, vol. 96, no. 5, pp. 1934–1951, Oct. 2006, doi: 10.1785/0120050143

[9] Y. Isbiliroglu, R. Taborda, and J. Bielak, ‘Coupled Soil-Structure Interaction Effects of Building Clusters During Earthquakes’, *Earthquake Spectra*, vol. 31, no. 1, pp. 463–500, Feb. 2015, doi: 10.1193/102412EQS315M.

[10] A. Wirgin, ‘Earthquakes in cities revisited’, *arXiv:1607.08856 [physics]*, Jul. 2016.

[11] P. Guéguen, P.-Y. Bard, and F. Chavez-Garcia, ‘Site-City Interaction in Mexico City-Like environments: An Analytical Study’, *Bulletin of the Seismological Society of America*, vol. 92, no. 2, pp. 794–811, 2002.

[12] C. Tsogka and A. Wirgin, ‘Simulation of seismic response in an idealized city’, *Soil Dynamics and Earthquake Engineering*, vol. 23, no. 5, pp. 391–402, Jul. 2003, doi: 10.1016/S0267-7261(03)00017-4.

[13] J. Groby, C. Tsogka, and A. Wirgin, ‘Simulation of seismic response in a city-like environment’, *Soil Dynamics and Earthquake Engineering*, vol. 25, no. 7–10, pp. 487–504, Aug. 2005, doi: 10.1016/j.soildyn.2004.11.007.

[14] P. Cacciola and A. Tombari, ‘A stochastic ground motion model for the urban environment’, *Probabilistic Engineering Mechanics*, p. 103026, Feb. 2020, doi: 10.1016/j.probengmech.2020.103026.

[15] J. P. Wolf, *Dynamic soil-structure interaction*. Englewood Cliffs, N.J.: Prentice-Hall, 1985.

[16] Y. Hisada, ‘An efficient method for computing Green’s functions for a layered half-space with sources and receivers at close depths (part 2)’, *Bulletin of the Seismological Society of America*, vol. 85, no. 4, pp. 1080–1093, Aug. 1995.

[17] Y. Hisada, ‘An efficient method for computing Green’s functions for a layered half-space with sources and receivers at close depths’, *Bulletin of the Seismological Society of America*, vol. 84, no. 5, pp. 1456–1472, Oct. 1994.

[18] P. M. Morse and K. U. Ingard, *Theoretical acoustics*. Princeton, N.J: Princeton University Press, 1986.

[19] H. G. Poulos, ‘Analysis of the Settlement of Pile Groups’, *Géotechnique*, vol. 18, no. 4, pp. 449–471, Dec. 1968,

[20] R. Dobry and G. Gazetas, ‘Simple method for dynamic stiffness and damping of floating pile groups’, *Geotechnique, London*, vol. 38, no. 4, pp. 557 – 574, 1988.

[21] P. C. Jennings and J. Bielak, ‘Dynamics of building-soil interaction’, *Bulletin of the Seismological Society of America*, vol. 63, no. 1, pp. 9–48, Feb. 1973.

[22] P. Cacciola, ‘A stochastic approach for generating spectrum compatible fully nonstationary earthquakes’, *Computers & Structures*, vol. 88, no. 15–16, pp. 889–901, Aug. 2010, doi: 10.1016/j.compstruc.2010.04.009.